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A Bilevel Approach for Optimal Price-Setting of Time-and-Level-of-Use Tariffs

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***Abstract*—The Time-and-Level-of-Use (TLOU) system is a recently developed approach for electric energy pricing, extending Time-of-Use with an energy capacity that customers can book in advance for a given consumption time. We define a bilevel optimization model for determining the pricing parameters of TLOU, maximizing the supplier revenue while anticipating an optimal reaction of the customer. A solution approach is built, based on the discrete finite set of optimality candidates of the lower-level customer problem.**

***Index Terms*—Demand response, electricity pricing, bilevel optimization, Time-and-Level-of-Use (TLOU)**

I. Introduction

With the increasing proportion of solar and wind generation in the energy mix, power systems have to accommodate greater variability on the supply side, along with more complex decisions on the demand side with generation and storage units down to the residential level. Demand Response (DR) has been seen as one of the promising approaches to these challenges, leveraging customers' flexibility to provide services for better operation of power systems. DR programs are often classified as incentive-based or price-based programs [2]. Different price-based programs are compared in [3] from the perspective of a supplier designing the pricing program, anticipating in a bilevel framework the reaction of a prosumer with storage and shifting capacity. A detailed review of the literature on bilevel optimization for price-based DR is available in [1]. Incentive-based programs require more commitment from the demand side, thus are often less suitable for targeting residential customers. Price-based DR requires less commitment and constraints on the customer side, thus offering a greater flexibility, but does not provide the supplier with strong guarantees on the actual or even expected demand. An approach taken in price-based DR is to offer varying reliability of electricity to customers [4], leaving the supplier free to adapt the power effectively served to the customer. The insufficiency of short-term estimation methods was identified in a technical report [5] as as one of the critical barriers to the effective implementation of DR. Even though Time-and-Level-of-Use (TLOU) is more related to price-based DR, the self-determined capacity creates an incentive for respecting the upper bound on the consumption on the part of the customer. It was defined in [6] as an extension of the Time-of-Use (TOU) pricing scheme, targeting specifically the issue with current large-scale DR programs identified in the FERC report [5]. Specifically, in the TLOU context, TOU is just a special setting with a null booked capacity. The authors of [6] develop the optimal planning and operation of a smart building under TLOU pricing. In this work, we propose a bilevel optimization model to assist in determining the optimal TLOU pricing structure for a supplier.

This work was presented at the 13th EUROGEN conference on Evolutionary and Deterministic Computing for Industrial Applications [1].

The reaction of the customer to the proposed pricing is integrated in the supplier decision problem, thus turning the customer-supplier interaction into a Stackelberg game solved as a bilevel optimization problem. Using specific properties of the customer problem, the optimal capacity decision can be reduced from a continuous set to a discrete finite number of choices which can be computed independently of other decisions. Through this transformation, the necessary and sufficient conditions for lower-level optimality are expressed as a set of linear constraints. For each time frame and corresponding consumption distribution, the set of optimal pricing options can be computed as solutions to the bilevel problem with fixed lower level. These options can be computed in advance, and one can be selected by the supplier ahead of the consumption time to create an incentive for the customer to book and consume a given capacity.

The contributions of this letter are the following: developing further the conceptual basis of the TLOU pricing and some of its key properties, building a bilevel model for the supplier's problem and a specialized solution method, and highlighting through numerical experiments the ability of TLOU to create different incentives depending on the supplier's needs.

This letter is structured as follows. Section II introduces the TLOU pricing, along with the variant used in this work. In Section III, the model of the supplier decision problem is developed, and necessary optimality conditions are defined to design an efficient solution method. Computational experiments are presented in Section IV for a supplier offering prices to incentivize the customer to follow a certain capacity profile across the day. Section V concludes the work.

II. TLOU pricing

The TLOU policy extends TOU by allowing a customer to book an energy capacity that they self-determine at each time frame depending on their planned requirements. Through this mechanism, they provide the supplier with information on the energy they would possibly consume. The capacity is the energy booked by the customer for a given time frame, following the same terminology as the initial description of TLOU in [6]. As in TOU, the price of energy depends on the time frame within the day, but also on the capacity booked by the customer. TLOU is applied in a three-phase process:

1) The supplier sends the pricing information $(K, \pi^L(c), \pi^H(c))$ to the customer.
2) The customer books a capacity from the supplier for the time frame before a given deadline.
3) After the time frame, the energy cost is computed depending on the energy consumed x_t and booked capacity c_t:
   - If $x_t \leq c_t$, then the applied price of energy is $\pi^L(c_t)$ and the energy cost is $\pi^L(c_t) \cdot x_t$.
   - If $x_t > c_t$, then the applied price of energy is $\pi^H(c_t)$ and the energy cost is $\pi^H(c_t) \cdot x_t$.

The first step corresponds to the pricing decision of the supplier for a given time frame. They then send the pricing to the customer, who takes in a second time their decision by booking a capacity $c_t \geq 0$, minimizing their expected cost. The pricing system is described by three elements: a booking fee K, a step-wise decreasing function $\pi^L(c_t)$ representing the lower energy price and a step-wise increasing function $\pi^H(c_t)$ representing the higher energy price. $\pi^L(c_t)$ will refer to the function of the capacity and π^L_j to the value of the lower price at step j.

An important feature of this DR program is the minimal information that must be exchanged by the parties involved. Unlike DR programs which require users to provide their projected needs or delegate the scheduling decisions to the supplier, TLOU only requires a capacity from the user, with the option of falling back on TOU pricing by not booking a capacity for any time frame.

We use the TLOU definition presented in [1]. Unlike the original convention of [6], the totality of the energy consumed is paid at the lower tariff if it remains below the booked capacity, and at the higher tariff otherwise, as described in Equation (1). In other words, if the consumption over the time frame remains below the booked capacity, the effective energy price is given by the lower tariff curve; if the consumption exceeds the booked capacity, the energy price is given by the higher tariff. This asymmetry in the pricing system creates a strong incentive to make the capacity act as an upper bound on the consumption, while under-consumption is penalized by a soft term proportional with the deviation as highlighted in Figure 2. Customers are still able to consume above the capacity if necessary, and the supplier is compensated for this deviation by the higher price that applies. The total cost for the customer associated with a booked capacity c and a consumption X_t for a time frame t is:

$$\mathcal{C}(c_t; X_t) = \begin{cases} K \cdot c_t + \pi^L(c_t) \cdot X_t, & \text{if } X_t \leq c_t, \\ K \cdot c_t + \pi^H(c_t) \cdot X_t & \text{otherwise.} \end{cases} \quad (1)$$

In the rest of this letter, the index of the considered time frame is dropped when not necessary in an expression to keep the notation succinct.

Proposition II.1. *If a customer books a capacity $c > 0$ for a given time frame and assuming $K + \pi^L(c) < \pi^H(c)$, the lowest cost per $kW \cdot h$ is reached when the consumption is exactly equal to the booked capacity.*

Proof. If $X \leq c$, the total cost is given by $Kc + X\pi^L(c)$, hence the relative cost per consumed unit of energy is $\frac{Kc}{X} + \pi^L(c)$, which is strictly decreasing with X. The discontinuity at $X = c$ is positive, since the relative total cost changes from $\frac{Kc}{X} + \pi^L(c)$ to $\frac{Kc}{X} + \pi^H(c)$. The relative cost at $X = c$ is $K + \pi^L(c)$, assuming $K + \pi^L(c) < \pi^H(c)$, there is no decrease of the relative cost below the point it reaches at $c = X$. □

An example illustrating Proposition II.1 is given Figure 1 and 2.

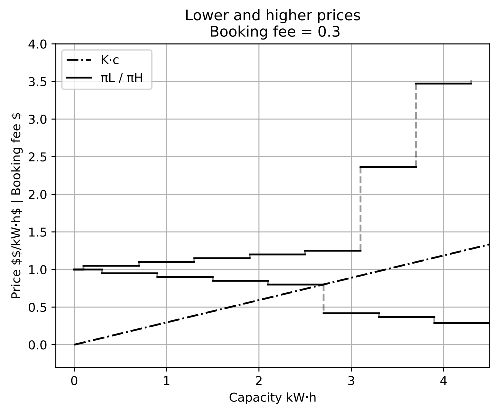

Figure 1. Example of TLOU pricing

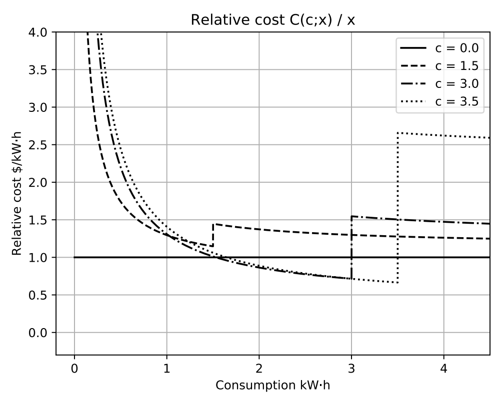

Figure 2. Relative cost of energy vs consumption for different capacities

One property of interest derived from Proposition II.1 is that the customer does not have any incentive to signal a capacity different from their consumption intent. This is in particular valuable for consuming units which are able to adjust their consumption through storage or flexible loads.

Another property of the proposed pricing is that the customer does not need to explicitly signal to the supplier that they do not wish to participate in TLOU at some given time frame. A customer can opt-out of the program simply by booking a capacity $c = 0$, for which the applied pricing matches TOU.

In a realistic setting, the supplier will have multiple, potentially heterogeneous customers. The model presented here tackles the single-customer case, but also applies when the supplier is able to offer different price settings specific to each individual customer.

III. Bilevel model for the supplier decision

In this section, we present a mathematical optimization problem modeling the supplier decision when setting the parameters of the TLOU pricing. The sequential decision process, with one agent reacting to the decision of the other and taking it into account when solving their decision problem, is represented as a Stackelberg game, and can be modeled as a bilevel optimization problem. In this bilevel formulation, the upper-level represents the energy supplier deciding on the pricing components (K, π^L, π^H), while the lower-level represents the customer's response to the supplier's decision, in terms of booking a capacity c.

We consider the consumption of the customer to be unknown to both the supplier and themselves when taking both the pricing and the booking decision. Both players know the probability distribution of this consumption ahead of time before making their decision. This probability distribution is assumed to be discrete and its support Ω is a finite set, with each element representing a consumption scenario $\omega \in \Omega$, with associated probability and value p_ω, x_ω respectively. The probability distribution and its support likely depend on the time frame considered t, we note it Ω_t when the time frame is specified. The expected cost of the customer, which is equivalent to the expected revenue of the supplier, is given as a function of both capacity and pricing:

$$\mathcal{C}(c, K, \pi^L, \pi^H) = K \cdot c + \sum_{\omega \in \Omega^-(c)} x_\omega p_\omega \pi^L(c) + \sum_{\omega \in \Omega^+(c)} x_\omega p_\omega \pi^H(c), \tag{2}$$

with any capacity booked defining a partition of the set of scenarios:

$$\Omega^-(c) = \{\omega \in \Omega, x_\omega \leq c\} \;\&\; \Omega^+(c) = \{\omega \in \Omega, x_\omega > c\}. \tag{3}$$

[1, Proposition 3.1] defines the subset of capacities respecting the necessary optimality conditions:

$$S_t = \{0\} \cup C^L \cup \Omega_t \tag{4}$$

where the steps of the lower and higher price functions are given at different breakpoints:

$$\begin{aligned} &\{c_0^L, c_1^L, c_2^L, ...\} = C^L \qquad \& \ \{\pi_0^L, \pi_1^L, \pi_2^L, ...\} = \pi^L \\ &\{c_0^H, c_1^H, c_2^H, ...\} = C^H \qquad \& \ \{\pi_0^H, \pi_1^H, \pi_2^H, ...\} = \pi^H \end{aligned}$$

The customer books the cost-minimizing capacity at each time frame, given the corresponding probability distribution. With the finite set of optimal candidates S_t, this constraint can be re-written as:

$$\mathcal{C}(c_t) \leq \mathcal{C}(c) \ \forall c \in S_t, \tag{5}$$

which corresponds to finitely many linear constraints. If multiple values of c reach a minimum cost, there is no unique choice the supplier can anticipate from the customer. In such situation, the supplier cannot correctly foresee the decision of the customer. They would want to ensure that the preferred solution of the customer is unique, by making one solution strictly lower in cost than all other capacity candidates. We re-formulate this requirement as the cost being lower than that of any other solution by at least a quantity $\delta > 0$. This quantity can be interpreted as the conservativeness of the customer (unwillingness to move to an optimal solution up to a difference of δ). It is a parameter of the decision-making process of the supplier, estimated a priori by the supplier based on its risk aversion and estimation of customer reactivity. Given this conservativeness parameter, the lower-level optimality conditions of a candidate $k \in S_t$ for a time frame t become:

$$\mathcal{C}(c_{tk}, K, \pi^L, \pi^H) \leq \mathcal{C}(c_{tl}, K, \pi^L, \pi^H) - \delta \ \forall l \in S_t \backslash \{k\}. \tag{6}$$

Given that the special case $c = 0$ matches the TOU pricing, the net difference in expected cost of δ is also ensuring that the effort of the customer committing to a capacity and engaging in the program should not be expected by the supplier below a net gain of δ for the customer.

The space of price parameters can be further restricted to include regularity constraints on the pricing curves, lower and upper bounds on the prices. All these constraints can be expressed as linear inequalities, and are summarized with the notation:

$$(K, \pi^L, \pi^H) \in \Phi \tag{7}$$

For each capacity candidate $k \in S_t \backslash \{0\}$, the supplier finds the optimal pricing parameters such that the candidate is the optimal capacity to book for the customer, and is at least better than any other candidate by a difference of δ. The k^{th} price-setting problem is expressed as:

$$\max_{K, \pi^L, \pi^H} \mathcal{C}(c_{tk}, K, \pi^L, \pi^H) \tag{8a}$$

$$(K, \pi^L, \pi^H) \in \Phi \tag{8b}$$

$$\mathcal{C}(c_{tk}, K, \pi^L, \pi^H) \leq \mathcal{C}(c_{lt}, K, \pi^L, \pi^H) - \delta \ \forall l \in S_t \backslash k, \tag{8c}$$

where $\mathcal{C}(c_{tk}, K, \pi^L, \pi^H)$ is defined in Equation (2). In [1], a multi-objective version of the same model is developed, the second objective is experimentally found to be non-conflicting with the revenue maximization.

Formulation (8) fully captures the bilevel nature of the problem through Constraint (8c), which specifies that the specific k-th capacity candidate must be the optimal choice by at least δ for the customer (lower level). Given the discrete nature of the lower- level decision (choosing the lowest-cost capacity candidate among a discrete set), the Karush-Kuhn-Tucker primal-dual optimality conditions could not be used. Constraint (8c) is used as an optimality condition for the lower-level, leveraging the fact that the optimality candidates can all be known prior to the optimization phase.

IV. Computational experiments

In this section, we present the setup and results for experiments with a supplier setting a TLOU pricing for 24 time frames, picking the pricing minimizing the deviation from the hourly. The data used and further experimentation are presented in [1, Section IV].

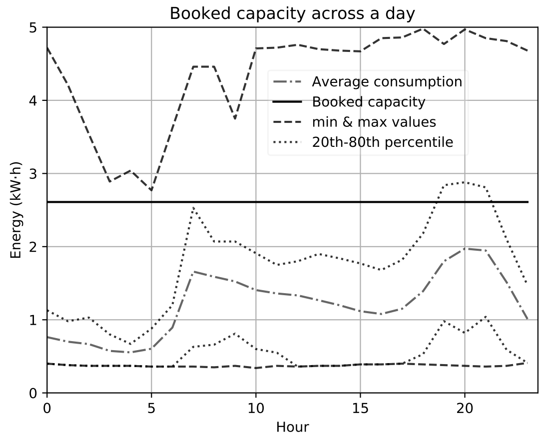

Figure 3. Booked capacity profile and load distribution bounds

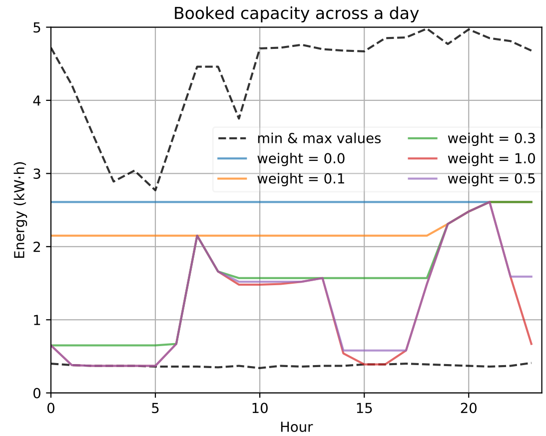

Figure 4. Booked capacity profile with the weighted bi-objective model

Given the probability distribution of the consumption at each time frame, the supplier can construct a TLOU price setting for each capacity candidate by solving problem 8. Once these hourly options have been computed, the supplier can choose to create an incentive for the customer to follow a profile. To avoid unexpected deviations, the supplier can set the prices, as to have a non-zero capacity as close as possible to the average consumption of each time frame, as shown in Figure 3. We use a $\delta = \$0.005$ and similar settings as [1, Section IV] for other parameters (with a reference TOU price of $\$1.0$).

Alternatively, a major objective of many DR programs is to smooth the demand curve by peak shaving and valley filling. The sum of variations from one time frame to the next can be minimized with the following linear optimization model:

$$\min_{v,z} \sum_{t=1}^{|T|-1} v_t \tag{9a}$$

$$v_t \geq \sum_{k\in S_t} c_t^k z_t^k - \sum_{k\in S_t} c_{t+1}^k z_{t+1}^k \quad \forall t \in \{1..|T|-1\} \tag{9b}$$

$$v_t \geq \sum_{k\in S_t} c_{t+1}^k z_{t+1}^k - \sum_{k\in S_t} c_t^k z_t^k \quad \forall t \in \{1..|T|-1\} \tag{9c}$$

$$\sum_{k\in S_t} z_t^k = 1 \qquad \forall t \in T \tag{9d}$$

In this model, $z_t^k = 1$ is equivalent to the supplier choosing the k^{th} candidate for the time frame t. v_t takes as value the absolute error between the capacity at t and $t+1$. The optimum of this model corresponds to a constant booked capacity over all the time frames. A trade-off can be set between this objective and the capacity averaged over each time frame:

$$\sum_{t=1}^{|T|} \sum_{k\in S_t} c_t^k z_t^k \;/\; |T|. \tag{10}$$

Different solutions corresponding to various trade-off are obtained by solving problem (11) with a weight w varying as shown in Figure 4. The weight parameter is the weight assigned to the average booked capacity, while the weight of the sum of variations remains at 1.

$$\min_{v,z} \sum_{t=1}^{|T|-1} v_t + w \sum_{t=1}^{|T|} \sum_{k\in S_t} c_t^k z_t^k \;/\; |T| \tag{11a}$$

$$\text{s.t} \quad (9b - 9d) \tag{11b}$$

V. Conclusion

In this letter, we present a bilevel model for a supplier optimizing the TLOU pricing, selecting for each capacity candidate the revenue-maximizing price setting. We illustrate the application with a selection of capacities across the day to smooth the capacity curve or to stay close to the average, and thus create an incentive for the customer to consume at this level. Future research will consider a single setting for multiple customers and exploiting continuous probability distributions of the consumption.

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